Correspondence

Propagation Constants in Rectangular Waveguide Partially Filled with Dielectric*

There is considerable current interest in the production of guided electromagnetic waves having phase velocities equal to or less than the speed of light in free space (for example, in the design of traveling-wave slot antennas and of devices involving electron traveling-wave interactions). Such phase velocities can be obtained conveniently by partially loading a rectangular waveguide with dielectric material. In antenna work particularly, because of the field configurations, it is usually desirable to place the dielectric interface so that it is parallel to the broad face of the waveguide, as indicated in Fig. 1. The calculation of phase velocities in such a waveguide has been considered in the literature,[1-3] and there is published information on some of the cutoff frequencies,[3] but (since in this case there is no convenient relationship between the cutoff frequencies and the propagation constants) there has been little detailed information available concerning the phase velocities as a function of waveguide proportions and dielectric material. Thus a compilation has been made of such information for the dominant (hybrid) mode.

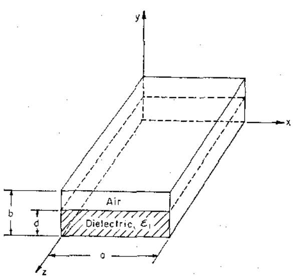

Fig. 1—Partially dielectric filled waveguide with dimension designations and coordinate system.

It can be shown[4] that the exact solution of Maxwell's equations, assuming perfect materials in the waveguide geometry indicated in Fig. 1, leads to the following transcendental equation for the propagation constants:

$$(k_{y1}/\epsilon_1) \tanh k_{y1}d = (k_{y2}/\epsilon_2) \tanh k_{y2}(d-b) \quad (1)$$

where k_{y1} and k_{y2} are the tranverse propagation constants related by

$$k_{y2}^2 = k_{y1}^2 + (2\pi/\lambda)^2(\epsilon_r - 1). \quad (2)$$

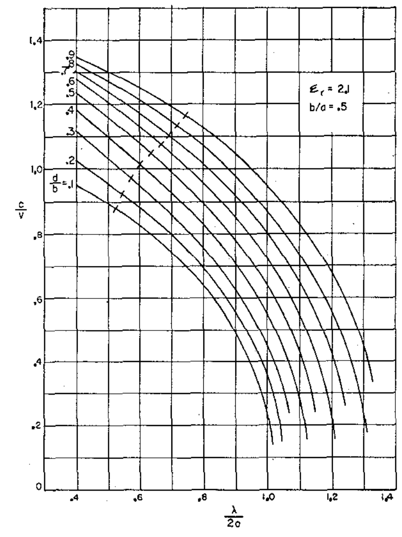

Fig. 2—Phase velocity of the dominant mode in the waveguide of Fig. 1, with permittivity 2.1 and aspect ratio of 0.5. Dashes across the curves indicate the region of cutoff of the next higher order mode.

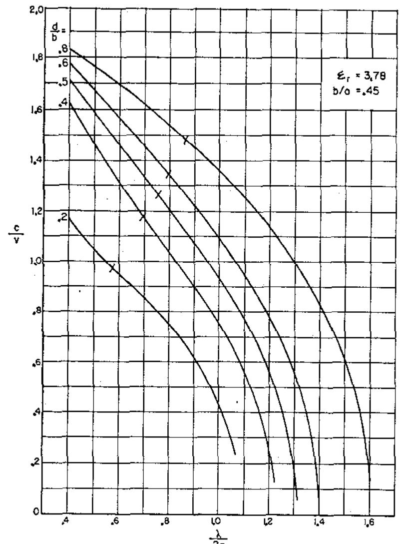

Fig. 3—Phase velocity of the dominant mode in the waveguide of Fig. 1, with permittivity 3.78 and aspect ratio of 0.45. Dashes across the curves indicate the region of cutoff of the next higher order mode.

The ratio of free space velocity, c, to the phase velocity in the waveguide, v, can be found from

$$c/v = \left[\epsilon_r - \left(\frac{n\lambda}{2a}\right)^2 + (k_{y1}\lambda/2\pi)^2\right]^{1/2} \quad (3)$$

where λ is the free space wavelength, $\epsilon_r = \epsilon_1/\epsilon_2$.

The computational difficulty lies in the the fact that the quantities k_{y1} and k_{y2} depend upon the parameters ϵ_r, b, and d in such a way that a change in any one of these requires a new numerical solution of the transcendental equation (1). Thus, any extensive tabulation of results makes the use of a high-speed computing machine almost imperative. At the University of Illinois a digital computer was employed, first to solve the trancendental equation, and then to calculate the (c/v) ratios. A detailed graphical and tabular presentation of the results is available in a technical report for limited distribution.[4] In this report, results can be found which pertain to seven of the common solid dielectric materials (dielectric constants 1.6, 2.1, 2.54, 3.78, 5.75, 10, and 13.7), with fillings (d/b) ranging from 10 to 90 per cent, and aspect ratios (b/a) ranging from 0.25 to 1. The report also includes tables of values to facilitate calculations in which more accuracy is required than can be obtained from the graphs.

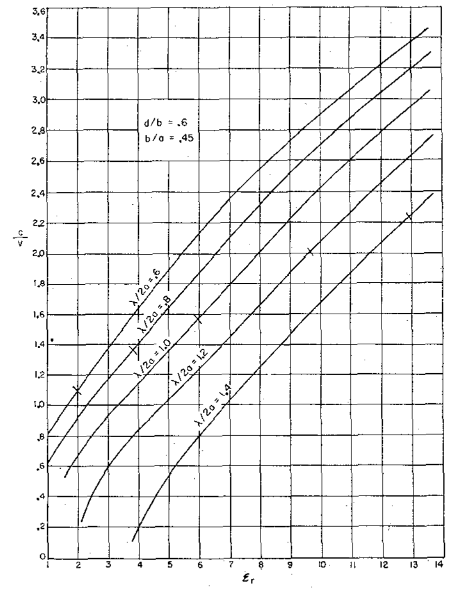

Fig. 4—Phase velocity ratio of the dominant mode in the waveguide of Fig. 1, as a function of permittivity, with 60 per cent filling and aspect ratio of 0.45. Cross lines indicate the region of cutoff of the next higher order mode.

For economy, this note includes only sample graphs of the type mentioned above. Figs. 2 and 3 show the variation of the phase velocity ratio (c/v) for waveguides having the proportions indicated, with dielectric constants 2.1 and 3.78, respectively. The dashes across the curves indicate the region of cutoff for whichever of the higher order modes has the lower cutoff frequency. The curves for the special cases of no filling and complete filling are of course the circles centered at the origin $(c/v = \lambda/2a = 0)$ which have radii of unity and $\epsilon_r^{1/2}$ (in units of $\lambda/2a$), respectively. Fig. 4 indicates another type of information which can be assembled from the data which are available. It exhibits the variation of c/v as a function of dielectric constant, for the particular waveguide proportions indicated.

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[1] L. Pincherle, "Electromagnetic waves in metal tubes filled longitudinally with two dielectrics," *Phys. Rev.*, vol. 66, pp. 118–130; September, 1944.

[2] N. Marcuvitz, "Waveguide Handbook," McGraw-Hill Book Co., Inc., New York, N. Y., pp. 391–393; 1950.

[3] J. Van Bladel and T. J. Higgins, "Cut-off frequency in two-dielectric layered rectangular waveguide," *J. Appl. Phys.*, vol. 22, pp. 329–334; March, 1951.

[4] W. L. Weeks, "Propagation Constants in Rectangular Waveguides Partially Filled with Dielectric," Antenna Lab., Elec. Eng. Res. Lab., University of Illinois, Urbana, Tech. Rep. No. 28; December, 1957.